Q.P. Code : 60862

Second Semester M.Sc. Degree Examination, July 2019

(CBCS Scheme)

Mathematics

Paper M 202 T – COMPLEX ANALYSIS

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates :

1) *Answer any **FIVE** questions.*
2) *All questions carry equal marks.*

1. (a) Define an analytic function and evaluate

$$\int_c \frac{z-3}{z^2+2z+5}\,dz \text{ where } C:|z+1-i|=2.$$

(b) State and prove the Cauchy's integral formula for derivatives and use it evaluate $\displaystyle\int_{|z|=1} \frac{\cos 2\pi z}{(2z-1)(z-3)}\,dz$

(c) State and prove Cauchy's theorem for a rectangle. (4 + 4 + 6)

2. (a) Define a zero of a function. Let $f(z)$ be analytic in a region D with zeros $a_1, a_2, \ldots\ldots, a_m$ repeated according to multiplicity. If γ is a simple closed curve in D which does not pass through any a_k, then prove that $\displaystyle\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}\,dz = \sum_{k=1}^{m} \eta(\gamma : a_k)$.

(b) State and prove Liouville's theorem. Deduce the fundamental theorem of algebra. (6 + 8)

3. (a) Define a power series and its radius of convergence. Find the radius of convergence of the following :

(i) $\displaystyle\frac{1}{2}.z + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + \ldots\ldots\ldots$

(ii) $\displaystyle 1 + \frac{a.b}{1.c}.z + \frac{a(a+1)b(b+1)}{1.2.c(c+1)}z^2 + \ldots\ldots$

(b) Let $f(z)=\sum_{n=0}^{\infty} a_n (z-a)^n$ in $\{|z-a|<R\}$ where R is the radius of convergence. Prove that the Taylor's series expansion of $f(z)$ in the neighborhood of a point 'a' is exactly the given power series.

(c) Find the Laurent's series expansion of $f(z)=\frac{z}{(z^2+1)(z^2+4)}$ in the regions

(i) $|z|<1$

(ii) $1<|z|<2$

(5 + 5 + 4)

4. (a) State and prove Taylor's theorem.

(b) Define an isolated singularity. Let $f(z)$ be analytic function having an isolated singularity at $z=a$. If $|f(z)|$ is bounded in a neighborhood $\{0<|z-a|<r\}$, then prove that $f(z)$ has a removable singularity at $z=a$.

(c) Define an isolated essential singularity. Prove that an analytic function comes arbitrarily close to any complex number in the neighborhood of an isolated essential singularity. (6 + 4 + 4)

5. (a) Define a pole and a residue. State and prove the Cauchy's residue theorem. (4)

(b) Evaluate any **TWO** of the following :

(i) $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2}$, $0<p<1$

(ii) $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$

(iii) $\int_{-\infty}^{\infty} e^{-x^2}\cos(2mx)\,dx$, $m>0$. (5 + 5)

6. (a) State and prove the argument principle theorem.

(b) Show that all the roots of $p(z)=z^8+4z^3+10$ lie between $1\le|z|\le 2$.

(c) Let $f(z)$ be a non-constant analytic function in a region D of the complex plane. Then prove that $|f(z)|$ has no maximum in D. (4 + 4 + 6)

7. (a) Define a convex function. State and prove the Hadamard's three circle theorem. Prove that $\log M(r)$ is a convex function of $\log r$.

(b) State and prove Weirstrass factorization theorem.

(8 + 6)

8. (a) State and prove Phragmen-Lindelof theorem.

(b) Derive the Poisson integral formula with standard notation. (7 + 7)

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